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STRESS TRANSMISSION THROUGH DISORDERED MEDIA CONFINED IN SILO GEOMETRY

Granular materials are found everywhere around us. A fundamental understanding of the behaviour of these materials is bound to have profound economic benefits. To understand the macroscopic behaviour of granular media and its relationship with the microscopic properties are major objectives of granular mechanics. However, presently macroscopic equations are empiric because of the complexities at the microscopic level. Granular media exhibit properties that are different from those of solids, liquids and gas and much of their behavior has not been fully understood. In this paper a brief survey of Physics of confined granular media has been presented. This is accomplished by reviewing the experimental and theoretical work done to determine the properties of static granular media and concerning theories presented to explain the properties.

1. INTRODUCTION:

A system composed of many macroscopic grains having diameter larger than one micron is known as granular matter. For instance, sand, wheat, seed, rice, concrete, drug compound, coal, lava flows. These seemingly simple materials display quite complex behavior much of which much of which has not yet been satisfactorily explained. For instance, in segregation phenomenon (segregation particles with material properties separate into different regions) it is shown that increase in entropy does not play an important role here. While thermodynamics predict that jostling would lead to mixing [1]. Another example of counterintuitive behavior exhibited by granular media is in case of silo and sand pile problem. In case of silo the bottom pressure does not increase indefinitely with the increase in height of packing but saturates after certain height [2]. Similarly in sand pile the maximum weight is not found at the centre of pile rather it is distributed in cone shape.

The importance of research in granular materials lies in the fact that they are second most manipulated materials after water, if measured in tons, and have enormous industrial and geophysical applications. So any improvement in body of knowledge would obviously lead to many economic benefits. Due to the numerous engineering applications involving granular matter, research in this field has been continually performed by engineers [3]. However, the subject has found renewed interest in the physics community (as well as other communities) in the past two decades [4]. Two main subfields have been developed that of granular gases and statics or quasi-static dense granular media.

The behavior of dense granular matter, which is dominated by prolonged inter-particle contacts, has proven more difficult for modeling. Quasi-static properties of granular materials are commonly modeled by using elasto-plastic models [5].

1. Macroscopic properties of static granular media

Much of early interest to investigate the static properties of granular media was sparked by the sandpile model of self-organized critically [5], which describes the avalanching process. Below we will discuss the static pilings of cohesion less grains. For example, we would like to be able to describe how forces or stresses are distributed in these systems. As a matter of fact, this is not a simple issue as, for instance, two apparently identical sandpiles but prepared in different ways can show rather contrasted bottom pressure profiles.

1.1.1. Sand pile

When grains are poured on the surface of sandpile the slope increases until it attains some threshold value. After that the additional grains are roll down by inclined surface, reducing the slope to the angle of repose. Thus angle of repose can be interpreted as it is an angle between the surface of pile and the horizontal surface. It depends on many factors such as density, surface area and shapes of the particles, and the coefficient of friction of the material. The angle of repose is a macroscopic parameter. When sand is stacked on the top of sand heap two phenomena are observed. An exceptional property is the observation, that a pressure dip is found at the centre or apex of sand pile. This has generated great controversy, because otherwise the traditional description would predict maximum pressure at the apex. The phenomena depend strongly on the construction procedure. It is reported by vanel et al [6] that if a sand pile prepared from a point source then pressure dip is observed, while the one constructed by constant raining results to close to common observation a pressure maximum.

1.1.2. The silo

The silo is a storage device. The principle of a typical experiment set up used for investigation of granular material confined in a silo is depicted in figure 1. Consider a column filled with a certain mass of grains M_{fill}. If one measures the weight felt by the bottom plate of this silo, then a naive guess would be the pressure would increase with the height of filling. On the contrary the careful experiments have revealed that this weight at the bottom of silo is only a fraction of total filled mass known as apparent mass M_{app} [6,7]. In other words, the confining walls of the silo support a substantial part of the total mass of the grains.

Janssen presented a theory regarding the stress distribution in a silo [2]. It was assumed in the theory that (a) The horizontal stresses in the granular medium are proportional the vertical stresses, $\sigma_{x} = K\sigma_{z}$ where *K* is redirection parameter. (b) Another simplification was made concerning friction. It was assumed that friction between the walls of silo and grains have reached at the maximum and are in yield criterion. (c) The density of material is also considered constant over all depths.

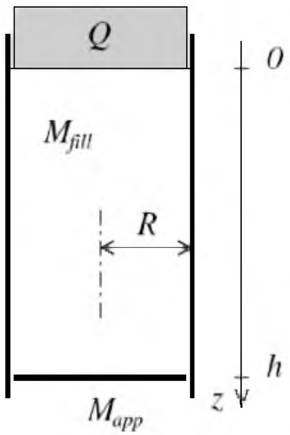

Figure 1 The sketch of a typical experimental set up of silo [8]

The mass measured at the bottom of cylinder is less than the filling mass called effective mass given by the equation

$$M_e = M_s(1 - e^{-M_f/M_s}) \qquad (1)$$

Where saturation mass is given by,

$$M_s = \rho\pi(D/2)^3/2\mu K \, . \qquad (2)$$

Here ρ represents the density of the material, D is the diameter of cylinder, μ is the coefficient of friction between the grains and the confining wall and λ is a central parameter in the theory known as characteristic length.

Ovarlez et al. [9] performed the same experiment with overload and no load conditions. The un-overloaded ($Q = 0$) data of Ovarlez et al. [9] are very well fitted by a relation like Eq. (1). Of course, the quality of such a comparison is crucially dependent on the experimental control of the packing density and the preparation procedure (which both govern the redistribution effect, i.e. the value of K), as well as the mobilization of the friction at the wall (i.e. the value of μ_w). In contrast, the presence of a finite overload Q is badly reproduced by the model. In particular, it predicts that M_{app} becomes independent of depth if this overload is precisely chosen such that $Q = M_{sat}$. This is not what is measured experimentally where an 'overshoot' is observed [7], Finally, it must be noted that no real 'granular features' are included in this approach. It is rather a model of screening effect. As a matter of fact, an elastic material confined into a rough rigid column would also show a saturation curve due to the Poisson effect which couples vertical and horizontal normal stresses. One can in particular compute the large-scale effective Janssen coefficient K in the framework of the linear isotropic elasticity. One gets $K = \nu$ and $K = \dfrac{\nu}{(1-\nu)}$ in two and three dimensions respectively (ν is as usual the Poisson ratio).

2. Microscopic Properties of Static Granular Media

The stresses inside a tall silo are transmitted via forces chains. This network of force chains is highly heterogeneous and anisotropic. The force chains are intense particularly along which the stresses are propagated [10]. It has been investigated by a robust experiment where a piece of carbon paper was placed at the bottom of pile. The size of mark left by each grains on the carbon paper was measured [10],

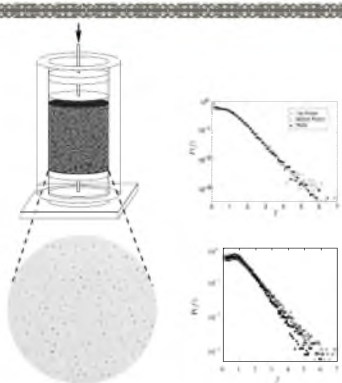

Figure 2 – shows the experimental set-up of the carbon paper.

Right: force distribution function *P(f)* [8].
The experimental data can be well fitted in functional form of *P(f)* as

$$P(f) \propto \left(f/\bar{f} \right)^{\alpha}, \text{ for } f < \bar{f}, \quad (3)$$

$$P(f) \propto e^{-\beta f/\bar{f}}, \text{ for } f > \bar{f} \quad (4)$$

Where $\bar{f}$ is mean value of contact force and α stays very close to zero. It is also found to positive and negative in experiments and simulation respectively, whereas the coefficient beta varies between one and two.

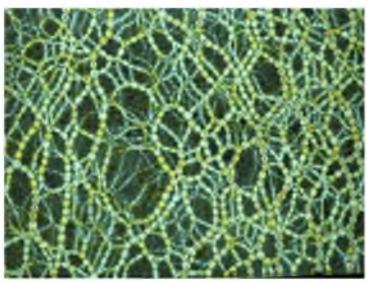

Figure 3 – shows how forces are propagated in granular materials [19].

Such behavior has been explained by a model known as the q-model. It predicts the propagation of stresses is distributed unevenly and randomly to the beads lying immediate below. This model is in good agreement with the experimental observed exponential distribution of large force chains [11]. The q model has some discrepancies, firstly being a scalar in nature only one component of stress has been taken care of. P. G. de Gennes solved this problem by introducing vectorial model, resulting in better prediction for force transmission. [20]. Secondly q model ido not predict the minimum of pressure at the centre of sand pile as observed in experiments. Similarly Janssen law is not reproduced here. (4) it appears that diffusive nature of the model is the main cause for these discrepancies. the saturation depth D_s where the stress distribution becomes independent of depth scales with the silo width R as $D_s \propto R^2$, at odds with the Janssen observation that predicts $D_s \approx A \cdot R$.

Therefore, Bouchaud et al. [12] put forwarded a more refined version of Janssen law. By introducing the assumption on the proportionality between horizontal and vertical stresses:

$$\sigma_{xx} = K\sigma_{zz} \quad (5)$$

$$\sigma_{yy} = K\sigma_{zz} \quad (6)$$

$$\sigma_{xy} = 0 \quad (7)$$

which lead to the linear equation:

$$\frac{\partial^2 \sigma_{zz}}{\partial z^2} - k\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\sigma_{zz} = 0 \quad (8)$$

The above equations being hyperbolic for vertical stresses differ from the elliptical equations for elastic medium [13]. From the q-model equation that is parabolic: it is equivalent to the equation for the wave propagation with Z as the ``time'' variable and K as the inverse of the propagation velocity. Using this model the minimum pressure measured experimentally at the bottom of sand pile has been reproduced [14]. In this connection another model introduced by Hemmingsson [15] also reproduces the minimum pressure at centre of heap and the correct Janssen model with the linear scaling.

Orientation of contacts

Another important microscopic quantity is the statistical orientation of the contacts. Getting insight into the how the contacts are oriented is a tedious job, because they are greatly influenced by gravity and external applied stress. In such cases a vivid anisotropy in contact orientation is observed. In two dimensional simulations carried out by Radjai et al [16] they created a layer of grains by allowing the grains to fall uniformly. In such configuration a compact packing has been achieved. It implies that contacts along the diagonal are more numerous than along horizontal and vertical ones. This is also attained in experiments [17, 18]. It is illustrated that large force are orientated along the main external stress while the smaller ones are distributed in more isotropic manner.

Another important finding was that although the large forces were less than 40%, however all the external stress was supported by them. The significant difference between the force probability distribution P(f) and the angular histogram of contact orientation Q(θ) is that the latter is very sensitive to the preparation of system. The contact orientation function is also called the "texture" and it provides the good representation of internal structure of the system.

3. Conclusion

In this paper only the distinctive properties of disordered media have been focused on. It is shown that the Physics of these media is still in infancy. However the Physics of granular materials include a wide variety of phenomenon having broad applications, ranging from powder to celestials objects. Consequently the experimental methods employed in this field are also diverse, ranging from carbon paper to positron emission tomography. Many of ideas devolved for these systems can be applied to various metastable systems where energy *KT* is unimportant, such as foams and superconducting vortex. It is expected that recent interest in research on granular media will lead to unravel new theories and experimental methods that would be helpful in technological processes but also augmenting knowledge regarding macroscopic and microscopic features of granular physics.

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Бункер геометриясында шектелген орта тәртіпсіздігі арқылы күш беру

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Передача напряжения через беспорядочные среды, ограниченные в геометрии бункера

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Біздің айналамызда сусымалы заттарды көруге болады. Бұл материалдар қылығының фундаменталды түсінігінің экономикалық пайдасы бар. Ұсақталған ортаның макроскопиялық мінезін және оның микроскопиялық қасиеттерімен байланысын түсіну үшін сусымалы орта механикасының мақсатын білу қажет. Бірақ та, қазіргі уақытта макроскопиялық теңдеулер микроскопиялық деңгейдегі қиындықтардың әсерінен эмпирикалық болып келеді. Сусымалы заттар газ, сұйық және қатты заттарда қарағанда озге қасиеттерге ие және олардың кобінің қылығы әлі зерттелмеген. Бұл мақалада шектелген сусымалы орта физикасына қысқа шолу берілген. Оған статикалық сусымалы орталардың қасиеттерін анықтауда эксперименталдық және теориялық талдау жасау бойынша жетеді.

Сыпучие материалы можно найти повсюду вокруг нас. Фундаментальное понимание поведения этих материалов, имеет большую экономическую выгоду. Чтобы понять, макроскопическое поведение гранулированных сред и его связь с микроскопическими свойствами необходимо знать основные цели механики сыпучих сред. Однако, в настоящее время макроскопические уравнения являются эмпирическими из-за сложностей на микроскопическом уровне. Сыпучие среды обладают свойствами, которые отличаются от твердых тел, жидкостей и газов, и большая часть их поведения не была полностью изучена. В данной статье представлен краткий обзор физики ограниченных сыпучих сред. Это достигается путем анализа экспериментальных и теоретических работ по определению свойств статических сыпучих сред.