ISSN 1811-1807

ҒЫЛЫМИ ЖУРНАЛ

С. Торайғыров атындағы
Павлодар мемлекеттік
университеті

ФИЗИКА-МАТЕМАТИКАЛЫҚ СЕРИЯ

4'2011

ПМУ ХАБАРШЫСЫ
ВЕСТНИК ПГУ

UDC 537.87

RELATED WAVES IN MAGNETIC MATERIALS

S.K.Tleukenov, A.B. Bilyalova, A.ZH. Samatova, M.A. Uaikhanova
Pavlodar State University named after S. Toraigyrov

Introduction

Magnetostrictive and piezoelectric crystals are widely used as acoustic electromagnetic and electromagnetic-acoustic commutators. The given work is devoted to the problem of interference of elastic and electromagnetic wave fields in anisotropic materials which have magnetostrictness and piezoelectricity effects with magnetoelectric effects. Despite the importance of this research, it should be said that this research is complicated, first of all, by many material parameters which baffle elastic, magnetostrictive, piezoelectric and magneto electric anisotropic features.

The effectiveness of applied commutators which are based on magnetostrictive and piezoelectric phenomena is not high, that is why all these phenomena in this work are taken up together with magnetoelectric in order to study the action of the effect.

The matrizer method is used as mathematic apparatus. The main advantage of this method is the uniformity of complicated physical and mechanical wave systems description. In this work the structure of fundamental solutions of interconnected system of Maxwell's equation and motion equation of elastic anisotropic medium together with magnetostrictive, piezoelectric and magnetoelectric effects is presented. There was a general analysis of elastic and electromagnetic wave fields. It was based on matrix structure of B coefficients and evident state of its elements. It was shown that the magneticelectric effect presence could lead to additional effects of this interaction. The further research of these effects is very important together with possibility to increase the efficiency of commutators (as it was said above). Besides all aforesaid, it is possible to study artificial heterostructure materials on the basis of the matrizer method; moreover these materials possess and unite many properties of totally different crystals.

1. Piezoelastic crystals. The spread of piezoelastic waves in anisotropic media of rhombic syngony of 222 class with magnetoelectric effects is studied on the basis of the analytical method of matrizer.

The analysis of waves diffusion for piezoelastic fields with the magnetoelectric effect is based on action equation of elastic media; these equations are to be solved together with Maxwell's equation.

Free energy is the sum of electromagnetic, elastic, magnetoelectric and piezoelastic components

$$F = F_{el} + F_{em} + F_{me} + F_{pe}$$

Densities of free energy are the following:

$$F_{el} = \frac{1}{2} c_{ijkl} \varepsilon_{ij} \varepsilon_{kl} \tag{1.1}$$

$$F_{el} = \frac{1}{2} c_{ijkl} \varepsilon_{ij} \varepsilon_{kl} \tag{1.2}$$

$$F_{mт} = Q_{ij} E_i H_j \tag{1.3}$$

$$F_{pe} = e_{ijk} \varepsilon_{ij} E_k \tag{1.4}$$

where c_{ijkl}- elastic parameters; ε_{ij}, μ_{ij}- components tensor of dielectric and magnetic permittivity; ε_{0}, μ_{0}- magnetic and electric constants; E_i, H_i - components of electrical and magnetic fields; e_{ijk}- piezoelectric parameters which connect the electric field with mechanical voltage; Q_{ij}- tensor components; the tensor is characterized by the magnetoelectric effect.

When the volume density of charge ρ and the vector of current density $\vec{j}$ are absent, the system of Maxwell's equations could be written in the following way:

$$rot\vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad rot\vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$div\vec{B} = 0 \qquad div\vec{D} = 0 \tag{1.5}$$

The system of equations (1.5) is considered together with action equation of elastic anisotropic medium:

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho \frac{\partial^2 u_i}{\partial t^2} \tag{1.6}$$

It could also define the proportion between voltage σ_{ij} and deformation ε_{kl}, which has an additional item connected with magnetic field influence. We get the proportion from differentiation of piezoelectric and elastic components of free energy (1.1),(1.4) according to ε_{ij}:

$$\sigma_{ij} = c_{ijkl} \varepsilon_{kl} - e_{ijk} E_k \tag{1.7}$$

$\varepsilon_{kl} = \frac{1}{2}(u_{l,k} + u_{k,l})$- deformation tensor; ρ- medium density;

u_i ,σ_{ij} - components of displacement vector and density tensor.

Material equations which reflect the qualities of the medium could be defined with the help of differentiation (1.2), (1.3), (1.4) according to E_i H_i and could be presented in the following way:

$$B_i = \mu_0 \mu_{ij} H_j + Q_{ij} E_j \tag{1.8}$$

$$D_i = \varepsilon_0 \varepsilon_{ij} E_j + e_{ijk} \varepsilon_{jk} + Q_{ij} H_j \tag{1.9}$$

The system of equations (1.5), (1.6) together with (1.7), (1.8), (1.9) could reduced to the equivalent system of equations of the first order:

$$\frac{d\vec{W}}{dz} = \hat{B}\vec{W};\vec{W} = (U_z,\sigma_{zz},U_x,\sigma_{xz},U_y,\sigma_{yz},E_y,H_x,H_y,E_x)^t \qquad (1.10)$$

where $\hat{B} = \hat{B}[c_{ijkl}(z), e_{ijk}(z), \varepsilon_{ij}(z), k_x, k_y]$ - matrix of coefficients, the elements of this matrix have the parameters of the medium where piezoelastic waves with the magnetoelectric effect are distributed.

In point (10) the matrix of coefficients $\hat{B}$ is the following:

$$\hat{B} = \begin{pmatrix}
0 & b_{12} & b_{13} & 0 & b_{15} & 0 & 0 & 0 & 0 & 0 \\
b_{21} & 0 & 0 & b_{24} & 0 & b_{26} & 0 & 0 & 0 & 0 \\
b_{31} & 0 & 0 & b_{34} & 0 & 0 & b_{37} & 0 & 0 & 0 \\
0 & b_{42} & b_{43} & 0 & b_{45} & 0 & b_{47} & b_{48} & b_{49} & b_{410} \\
b_{51} & 0 & 0 & 0 & 0 & b_{56} & 0 & 0 & 0 & b_{510} \\
0 & b_{62} & b_{63} & 0 & b_{65} & 0 & b_{67} & -b_{68} & b_{69} & b_{610} \\
0 & 0 & -\frac{\omega}{i}b_{73} & 0 & \frac{\omega}{i}b_{75} & 0 & b_{77} & b_{78} & b_{79} & b_{710} \\
0 & 0 & -\frac{\omega}{i}b_{83} & i\omega b_{84} & -\frac{\omega}{i}b_{85} & 0 & b_{87} & b_{88} & b_{89} & b_{810} \\
0 & 0 & \frac{\omega}{i}b_{93} & 0 & \frac{\omega}{i}b_{95} & -i\omega b_{96} & -b_{97} & -b_{98} & -b_{99} & b_{910} \\
0 & 0 & \frac{\omega}{i}b_{103} & 0 & \frac{\omega}{i}b_{105} & 0 & -b_{107} & -b_{108} & b_{109} & -b_{1010}
\end{pmatrix}, \qquad (1.11)$$

where b_{ij} – matrix components of coefficients for rhombic syngony of 222 class.

Evidently, the influence of the magnetoelectric effect is connected with coefficients b_{47}, b_{410}, b_{67}, b_{610}, b_{77}, b_{710}, b_{87}, b_{89}, b_{810}, b_{910}.

From point (1.11) we can see that the structures of matrix coefficients when the waves are distributing in coordinate space xz, (ky=0) are the following:

$$\hat{B} = \begin{pmatrix}
0 & b_{12} & b_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
b_{21} & 0 & 0 & b_{24} & 0 & 0 & 0 & 0 & 0 & 0 \\
b_{31} & 0 & 0 & b_{34} & 0 & 0 & b_{37} & 0 & 0 & 0 \\
0 & b_{42} & b_{43} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & b_{56} & 0 & 0 & 0 & b_{510} \\
0 & 0 & 0 & 0 & b_{65} & 0 & b_{67} & 0 & b_{69} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{78} & 0 & b_{710} \\
0 & 0 & 0 & i\omega b_{84} & -\frac{\omega}{i}b_{85} & 0 & b_{87} & 0 & b_{89} & 0 \\
0 & 0 & 0 & 0 & 0 & -i\omega b_{96} & 0 & -b_{98} & 0 & b_{910} \\
0 & 0 & 0 & 0 & \frac{\omega}{i}b_{105} & 0 & -b_{107} & 0 & b_{109} & 0
\end{pmatrix}, \qquad (1.12)$$

From the last structure of matrix coefficients we can see that extensional and transverse elastic waves as well as electromagnetic waves distributing in the medium are interconnected. The mutual transformation between elastic waves is defined by elements of electromagnetic waves b13, b24 , and b710, b89 define it between electromagnetic waves. The interconnection between elastic and electromagnetic waves is given by elements b37, b510, b67, b69. The influence of the electromagnetic effect is defined by elements b710, b89.

2. Piezomagnetic crystals. The analysis of waves distribution for magnetoelastic fields with magnetoeffects is based on elastic equations, solving together with Maxwell's equation.

Free energy is defined by electromagnetic, elastic, magnetoelectric and magnetoelastic components.

$$F = F_{el} + F_{em} + F_{me} + F_{pm},$$

here:

$$F_{el} = \frac{1}{2} c_{ijkl} \varepsilon_{ij} \varepsilon_{kl} \quad (2.1)$$

$$F_{em} = \varepsilon_0 \varepsilon_{ij} E_i E_j + \mu_0 \mu_{ij} H_i H_j \quad (2.2)$$

$$F_{me} = Q_{ij} E_i H_j \quad (2.3)$$

$$F_{pm} = \delta_{ijk} \varepsilon_{ij} H_k \quad (2.4)$$

Where c_{ijkl} - elastic hardness; ε_{ij}, μ_{ij} - components of tensor of dielectric and magnetic permittivity; ε_0, μ_0- magnetic and electric constants; E_{ij}, H_{ij}- components of electrical and magnetic fields; δ_{ijk}- Piezomagnetic parameters; Q_{ij}- tensor components; the tensor is characterized by the magnetoelectric effect.

The system of Maxwell's equations when the equation is equal to zero of volume density of charges and the vector of currents density is written in the following way:

$$rot\vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad rot\vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$div\vec{B} = 0 \qquad div\vec{D} = 0 \quad (2.5)$$

The system of equations (2.5) is considered together with action equation of elastic anisotropic media:

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho \frac{\partial^2 u_i}{\partial t^2} \quad (2.6)$$

they also define the proportion between voltage σ_{ij} and deformation ε_{kl}, which has an additional item connected with magnetic field influence. This proportion could be achieved from the differentiation of piezomagnetic and elastic energy components according to ε_{ij} :

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} + \delta_{ijk}H_k \tag{2.7}$$

where $\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ - deformation tensor; ρ - medium density; u_i, σ_{ij} - components of displacement vector and density tensor.

Material equations which reflect the qualities of the medium could be presented in the following way:

$$B_i = \mu_0\mu_{ij}H_j + Q_{ij}E_j + \delta_{ijk}\varepsilon_{jk} \tag{2.8}$$

$$D_i = \varepsilon_0\varepsilon_{ij}E_j + Q_{ij}H_j \tag{2.9}$$

The system of equations (2.5) – (2.6) together with (2.7),(2.8),(2.9) is reduced to equivalent system of first order equation:

$$\frac{d\vec{W}}{dz} = \hat{B}\vec{W}; \vec{W} = (U_z, \sigma_{zz}, U_x, \sigma_{xz}, U_y, \sigma_{yz}, E_x, H_x, H_y, E_y)^T \tag{2.10}$$

where $\hat{B} = \hat{B}[c_{ijkl}(z), \delta_{ijk}(z), э_{ij}(z), k_x, k_y]$ - matrix of coefficients, the coefficients of the matrix have the medium parameters where magnetoelastic waves with the magnetoelectrical effect are distributing.

If we pay our attention to magnetoelastic waves in point (10), we will see that matrix of coefficients $\hat{B}$ is following:

$$\hat{B} = \begin{pmatrix} 0 & b_{12} & b_{13} & 0 & b_{15} & 0 & 0 & 0 & 0 & 0 \\ b_{21} & 0 & 0 & b_{24} & 0 & b_{26} & 0 & 0 & 0 & 0 \\ b_{31} & 0 & 0 & b_{34} & 0 & 0 & 0 & 0 & b_{39} & 0 \\ 0 & b_{42} & b_{43} & 0 & b_{45} & 0 & b_{47} & b_{48} & b_{49} & b_{410} \\ b_{51} & 0 & 0 & 0 & 0 & b_{56} & 0 & b_{58} & 0 & 0 \\ 0 & b_{62} & b_{63} & 0 & b_{65} & 0 & b_{67} & -b_{68} & b_{69} & b_{610} \\ 0 & 0 & \frac{\omega}{i}b_{73} & 0 & -\frac{\omega}{i}b_{75} & -i\omega b_{76} & b_{77} & b_{78} & b_{79} & b_{710} \\ 0 & 0 & \frac{\omega}{i}b_{83} & 0 & \frac{\omega}{i}b_{85} & 0 & b_{87} & b_{88} & b_{89} & b_{810} \\ 0 & 0 & -\frac{\omega}{i}b_{93} & 0 & -\frac{\omega}{i}b_{95} & 0 & -b_{97} & -b_{98} & -b_{99} & b_{910} \\ 0 & 0 & -\frac{\omega}{i}b_{103} & i\omega b_{104} & -\frac{\omega}{i}b_{105} & 0 & -b_{107} & -b_{108} & b_{109} & -b_{1010} \end{pmatrix} \tag{2.11}$$

where b_{ij} – components of the matrix coefficients for rhombic syngony of 222 class.

As we can see the influence of the magnetoelectric effect is connected with coefficients b_{48}, b_{49}, b_{69}, b_{77}, b_{710}, b_{89}.

From (2.11) the structure of matrix coefficients are followed together with waves distribution in coordinate spaces xz (ky=0):

$$\hat{B}=\begin{pmatrix} 0 & b_{12} & b_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ b_{21} & 0 & 0 & b_{24} & 0 & 0 & 0 & 0 & 0 & 0 \\ b_{31} & 0 & 0 & b_{34} & 0 & 0 & 0 & 0 & b_{39} & 0 \\ 0 & b_{42} & b_{43} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & b_{56} & 0 & b_{58} & 0 & 0 \\ 0 & 0 & 0 & 0 & b_{65} & 0 & b_{67} & 0 & b_{69} & 0 \\ 0 & 0 & 0 & 0 & 0 & -i\omega b_{11} & 0 & b_{78} & 0 & b_{710} \\ 0 & 0 & 0 & 0 & \frac{\omega}{i}b_{65} & 0 & b_{87} & 0 & b_{89} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -b_{710} & 0 & b_{910} \\ 0 & 0 & 0 & i\omega b_{m} & -\frac{\omega}{i}b_{65} & 0 & -b_{69} & 0 & b_{109} & 0 \end{pmatrix}, \quad (2.12)$$

From the last structure of matrix coefficients we can see that extensional and transverse elastic waves as well as electromagnetic waves distributing in the medium are interconnected. The mutual transformation between elastic waves is defined by elements of electromagnetic waves b_{13}, b_{24} and b_{710}, b_{89} define it between electromagnetic waves. The interconnection between elastic and electromagnetic waves is given by elements b_{39}, b_{58}, b_{67}, b_{69}. The influence of the electromagnetic effect is defined by elements b_{710}, b_{89}.

According to the matrix of the coefficients we can say that five waves are distributing: the elastic extensional wave which is connected with the elastic transverse wave of y-polarization and with electromagnetic TM-wave and the elastic transverse wave of x-polarization is connected with electromagnetic TE-wave and TM-wave. The connection between the wave of x-polarization and electromagnetic TE-wave and TM-wave is determined by coefficients b_{37}, b_{48}, b_{410}, and between y-polarization wave with TM-wave the connection is determined by coefficient b_{510}.

The matrix equation which is a system of ten usual differential equations of the first order with variable coefficients, has the thing we are looking for, and the main point in further researches is the equation. This equation describes harmonically timely dependent electroelastic waving processes in one heterogeneous material anisotropic media of rhombic syngony of 222 class with piezoelastic, magnetoelastic and magnetoelectric effects.

Matrix $\hat{B}$ is called a matrix of coefficients. The dependence between its elements defines its structure and this structure is called a structure of matrix coefficients.

REFERENCES

1. Вайнштейн Б.К. Современная кристаллография. Том 4 - Наука,1979.
2. Гантмахер Ф.Р. Теория матриц. - М.: Наука, 1988 - 522с.
3. Ландау Л.Д., Лифшиц Е.М. Теоретическая физика. Том 2 Наука, 1988.
4. Новацкий В. Теория упругости. - М.: Мир, 1986.
5. Тлеукенов С.К., Оспан А.Т., Изучение электромагнитных полей в анизотропных средах.- Алматы, 2001.

Түйіндеме

Жұмыста алғаш рет магнитті-электрлік эффектісі бар орталарда электр-серпімді және магнитті-серпімді толқындардың таралу теңдеулерінің фундаменталды шешімдерінің құрылымы жасалынды. Электромагниттік және серпімді толқындардың өзара әсерлесуінің талдауы келтірілді. Магнитті-электрлік эффекттің әсері көрсетілді.

Резюме

В работе впервые построена структура фундаментальных решений уравнений распространения электроупругих и магнитоупругих волн в средах с магнитоэлектрическим эффектом. Приведен анализ взаимодействия упругих и электромагнитных волн. Показано влияние магнитоэлектрического эффекта.